

Diquark mass differences from unquenched lattice QCD

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Abstract

We calculate diquark correlation functions in the Landau gauge on the lattice using overlap valence quarks and 2+1-flavor domain wall fermion configurations. Quark masses are extracted from the scalar part of quark propagators in the Landau gauge. Scalar diquark quark mass difference and axial vector scalar diquark mass difference are obtained for diquarks composed of two light quarks and of a strange and a light quark. Light sea quark mass dependence of the results is examined. Two lattice spacings are used to check the discretization effects. The coarse and fine lattices are of sizes $24^3 \times 64$ and $32^3 \times 64$ with inverse spacings $1/a = 1.75(4)$ GeV and $2.33(5)$ GeV, respectively.

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1 Introduction

Diquarks were introduced long time ago and are used in many phenomenology studies of strong interactions. For example, diquarks are used to describe baryons with a quark-diquark picture for explaining the missing states [1]. Also they are used to explain the $\Delta I = 1/2$ rule in weak nonleptonic decays [2, 3]. A review on diquarks are given in Ref. [1]. Since the discovery of $X(3872)$ by the Belle Collaboration [4], experiments have observed many so called XYZ states [5]. It is difficult to interpret these states as conventional heavy quarkonia. Molecules and tetraquarks are proposed to explain some of these quarkoniumlike states. In the tetraquark scenario, a diquark and an antidiquark form a four quark state.

Quantum Chromodynamics (QCD) is the theory describing the strong interaction among quarks and gluons, from which the properties of diquark correlations (if any) should be understood. At low energies, QCD has to be solved by nonperturbative methods since the strong coupling constant becomes so strong that perturbative calculations break down. In this work we study diquarks starting from the QCD action by using lattice QCD simulations. Diquarks have been studied on the lattice from various approaches [6, 7, 8, 9, 10, 11, 12] to deal with the fact that they are not color singlets. Diquark masses were also calculated in QCD sum rules, see, for example, Refs. [13, 14]. And the stability of diquarks were discussed as well [15].

The scalar diquark is supposed to be the state with the strongest correlation. The mass difference between diquarks with various quantum numbers can reflect the relative size of the correlation. In Ref. [16], scalar diquark and quark mass difference as well as the axial vector and scalar diquark mass difference are estimated from baryon spectroscopy. On the lattice, diquark mass and mass differences can be studied in a fixed gauge. So far, the masses and mass differences are calculated mostly in the quenched approximation on the lattice. Here we calculate them by using $2 + 1$ flavor domain wall fermion configurations. For the valence quark, we use overlap fermions. Previous lattice calculations focused on diquarks composed of the light up and down quarks. In this work we consider diquarks composed of a strange and a light quark as well as of two light quarks.

Diquark correlations are induced by spin dependent interactions. Thus they become weaker as the masses of quark increase. We can look into the mass dependence of diquark correlations by varying the current quark masses on the lattice.

Our results of the diquark mass difference and diquark quark mass difference are summarized in Tab. 14. In general they agree with the estimations in Ref. [16]. The exception is the scalar diquark and strange quark mass difference for diquarks composed of a strange and a light quark. Our result of this difference is smaller than the estimation in Ref. [16].

The paper is organized as follows. In section 2, we present the details of our calculation. The results and discussion are given in section 3. Finally we summarize in section 4.

Table 1: Interpolating fields and correlation functions of diquarks. A trace is performed in color space.

J^P (diquark)	Operators	Correlators
0^+ (good)	$J_c^5 = \epsilon_{abc}[q_1^a C \gamma_5 q_2^b]$	$\sum_{\vec{x}} \langle \Omega T J_c^5(x) \bar{J}_c^5(0) \Omega \rangle$
0^+ (good)	$J_c^{05} = \epsilon_{abc}[q_1^a C \gamma_0 \gamma_5 q_2^b]$	$\sum_{\vec{x}} \langle \Omega T J_c^{05}(x) \bar{J}_c^{05}(0) \Omega \rangle$
1^+ (bad)	$J_c^i = \epsilon_{abc}[q_1^a C \gamma_i q_2^b]$	$\frac{1}{3} \sum_i \sum_{\vec{x}} \langle \Omega T J_c^i(x) \bar{J}_c^i(0) \Omega \rangle$
1^+ (bad)	$J_c = \epsilon_{abc}[q_1^a C q_2^b]$	$\sum_{\vec{x}} \langle \Omega T J_c(x) \bar{J}_c(0) \Omega \rangle$
0^-	$J_c^I = \epsilon_{abc}[q_1^a C q_2^b]$	$\sum_{\vec{x}} \langle \Omega T J_c^I(x) \bar{J}_c^I(0) \Omega \rangle$
0^-	$J_c^0 = \epsilon_{abc}[q_1^a C \gamma_0 q_2^b]$	$\sum_{\vec{x}} \langle \Omega T J_c^0(x) \bar{J}_c^0(0) \Omega \rangle$
1^-	$J_c^{i5} = \epsilon_{abc}[q_1^a C \gamma_i \gamma_5 q_2^b]$	$\frac{1}{3} \sum_i \sum_{\vec{x}} \langle \Omega T J_c^{i5}(x) \bar{J}_c^{i5}(0) \Omega \rangle$

2 Correlation functions and lattice setup

The two quarks in the scalar diquark ($J^P = 0^+$) in the color antitriplet representation are attractive to each other as favored by perturbative one gluon exchange [17] and by instanton interactions [18, 19]. It is often called a “good” diquark. The next favored diquark is the axial vector diquark ($J^P = 1^+$) and is called a “bad” diquark. The other diquarks with $J^P = 0^-, 1^-$ are supposed to be even energetic and therefore their masses are higher. The diquarks in the color sextet representation have much larger color electrostatic field energy and are not favored by various models [16].

Therefore we focus on the color antitriplet diquarks and calculate their masses on the lattice. The interpolating operators used in this work for diquarks with various quantum numbers are given in Tab. 1, where C is the charge conjugation operator. In the operators q_1 and q_2 are u and $d(s)$ respectively. In calculating the two point functions in Tab. 1, we take a trace in color space and project to zero momentum. Diquarks are not color singlet. Their two point correlation functions have to be computed in a fixed gauge. We use the Landau gauge.

Alternatively, one can combine a diquark with an infinitely heavy quark (i.e. a Polyakov line) to get a color singlet state and calculate its correlation function. This is gauge invariant, however it may have path dependence [6].

We use the RBC-UKQCD configurations generated with 2+1-flavor domain wall fermions [20]. The parameters of the ensembles used in this work are given in Table 2. On the coarse lattice, two different light sea quark masses are used to check the sea quark mass dependence of our results. To examine finite lattice spacing effects, we use one ensemble on a fine lattice. To improve the signal, on each configuration on the coarse lattice we compute eight point source quark propagators. The sources are evenly located on eight time slices. On each time slice the source position is randomly chosen from one configuration to another to reduce data correlations. For the vector and axial vector diquarks, we average over the three directions ($i = 1, 2, 3$) to increase statistics. Also we average the data in the forward and backward time directions.

Table 2: Parameters of configurations with 2+1 flavor dynamical domain wall fermions (RBC-UKQCD). The residual masses are from Ref. [20]. The lattice spacings are from Ref. [24].

$1/a(\text{GeV})$	label	am_{sea}	volume	$N_{src} \times N_{conf}$	am_{res}
1.75(4)	c005	0.005/0.04	$24^3 \times 64$	8×92	0.003152(43)
	c02	0.02/0.04	$24^3 \times 64$	8×99	
2.33(5)	f004	0.004/0.03	$32^3 \times 64$	1×50	0.0006664(76)

Table 3: Overlap valence quark masses in lattice units used in this work.

$24^3 \times 64$	0.01350	0.02430	0.04890	0.06700	0.15000	0.33000	0.67000
$32^3 \times 64$	0.00677	0.01290	0.02400	0.04700	0.18000	0.28000	0.50000

For the valence quark, we use overlap fermions. The massless overlap operator [21] is defined as

$$D_{ov}(\rho) = 1 + \gamma_5 \varepsilon(\gamma_5 D_w(\rho)). \quad (1)$$

Here ε is the matrix sign function and $D_w(\rho)$ is the usual Wilson fermion operator, except with a negative mass parameter $-\rho = 1/2\kappa - 4$ in which $\kappa_c < \kappa < 0.25$. We use $\kappa = 0.2$ in our calculation that corresponds to $\rho = 1.5$. The massive overlap Dirac operator is defined as

$$\begin{aligned} D_m &= \rho D_{ov}(\rho) + m \left(1 - \frac{D_{ov}(\rho)}{2}\right) \\ &= \rho + \frac{m}{2} + \left(\rho - \frac{m}{2}\right) \gamma_5 \varepsilon(\gamma_5 D_w(\rho)). \end{aligned} \quad (2)$$

To accommodate the $SU(3)$ chiral transformation, it is usually convenient to use the chirally regulated field $\hat{\psi} = (1 - \frac{1}{2}D_{ov})\psi$ in lieu of ψ in the interpolation field and operators. This is equivalent to leaving the unmodified operators and instead adopting the effective quark propagator

$$G \equiv D_{eff}^{-1} \equiv \left(1 - \frac{D_{ov}}{2}\right) D_m^{-1} = \frac{1}{D_c + m}, \quad (3)$$

where $D_c = \frac{\rho D_{ov}}{1 - D_{ov}/2}$ is chiral, i.e. $\{\gamma_5, D_c\} = 0$ [22].

The overlap valence quark masses in lattice units are given in Tab. 3. Using the quark mass renormalization constants from Ref. [23] and the lattice spacings, one finds the corresponding $\overline{\text{MS}}$ quark masses at 2 GeV are from about 20 MeV to 1 GeV. The bare quark masses $am_q = 0.067$ and 0.047 on the coarse and fine lattices correspond to the physical strange quark mass respectively within our uncertainty [24]. Our largest quark mass is less than but close to the charm quark mass.

Table 4: Pion, nucleon and Delta masses from ensemble c005. $1/a = 1.75(4)$ is used to convert to physical units.

am_q	am_{ps}	m_{ps}/GeV	am_N	$am_{\Delta^{++}}$	$a(m_{\Delta^{++}} - m_N)$
0.01350	0.17911(46)	0.3134(72)	0.677(17)	0.818(41)	0.141(39)
0.02430	0.23670(43)	0.4142(95)	0.7218(61)	0.863(20)	0.141(19)
0.04890	0.33520(55)	0.587(13)	0.8043(64)	0.924(18)	0.120(18)
0.06700	0.39173(46)	0.686(16)	0.8643(47)	0.973(11)	0.109(12)
0.15000	0.60552(35)	1.060(24)	1.1247(42)	1.2049(66)	0.0802(72)
0.33000	0.97666(36)	1.709(39)	1.6398(22)	1.6991(27)	0.0593(26)

3 Results and discussion

In this section, we give the numerical results on all three ensembles given in Tab. 2. The statistical errors of our results are from bootstraps with 500 samples. On the coarse lattice, we have two ensembles with different light sea quark masses. Thus we only check the sea quark mass dependence in the results but do not try to extrapolate to the chiral limit of the sea quark. The results on the fine lattice, ensemble f004, are used to check the discretization effects but have relatively large uncertainty due to the limited statistics.

3.1 Pion, nucleon and Delta masses

For the convenience of chiral extrapolation later, we first give the pion masses from our data. We also compute the two point correlation functions for the nucleon and Δ^{++} baryon, from which we extract their masses at our pion masses. We use the usual interpolating operators $\bar{u}\gamma_5 d$, $\epsilon_{abc}[u^{Ta}(C\gamma_5)d^b]u^c$ and $\epsilon_{abc}[u^{Ta}(C\gamma_\mu)u^b]u^c$ for these hadrons respectively. The correlation functions are projected to zero 3-momentum and to positive parity (for the baryons). At large t , the ground state dominates and the hadron mass is obtained from a single exponential fit to the correlator.

The numerical values of these hadron masses are given in Tabs. 4, 5, 6 for ensemble c005, c02 and f004 respectively. On ensemble f004, the signal to noise ratio for the correlators of the Delta baryon is too bad for us to obtain its mass, especially at light quark masses. At leading order in heavy baryon chiral perturbation theory [26], the nucleon mass is given by

$$M_N = M_0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3. \quad (4)$$

Here we take the experiment values 1.267 for g_A and 92.4 MeV for f_π to fit our nucleon mass as a function of the pion mass. Note by doing this, we have ignored the effects from the mixed action setup. Half of our pion masses are larger than 600 MeV. Thus we do not expect the pion mass dependence of all the baryon masses can be well described by formulae from chiral perturbation

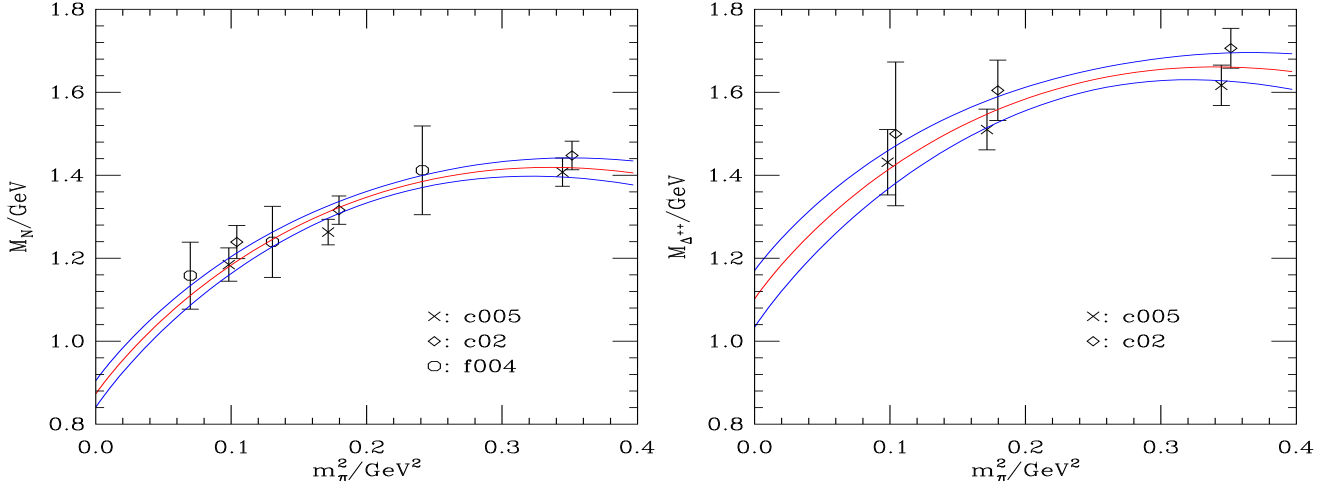


Figure 1: Left: The chiral fit to the nucleon mass using data from all three ensembles at pion mass less than 600 MeV. Right: The chiral fit to the mass of Δ^{++} using data from ensembles c005 and c02 at pion masses less than 600 MeV.

theory. In using Eq.(4) to do the fit, we only include the data points with pion mass less than 600 MeV.

In the left graph of Fig. 1, the nucleon masses from all three ensembles are plotted against the pion mass squared. Here and in the rest of the paper, we have taken into account the uncertainties of the lattice spacings in converting our results into physical units. We do not see big sea quark mass dependence or discretization effects. The two parameter (M_0 and c_1) fit is shown by the red curve and has a χ^2 per degree of freedom (dof) equal to 0.73. At the physical pion mass 140 MeV, the fit gives $m_N = 953(30)$ MeV, which agrees with the experimental value 940 MeV. A three parameter function $M_N = M_0 + c_1 m_\pi^2 + c_2 m_\pi^3$ can also fit the same data with a good χ^2 . It gives a nucleon mass (1.09(9) GeV) at the physical point. This is similar to the chiral fits in Refs. [27, 28].

For the pion mass dependence of the Delta baryon, the leading one loop result from heavy baryon chiral perturbation theory has a same form as in Eq.(4) if the SU(6) relation $H_A = 9g_A/5$ is used for the Delta baryon axial coupling H_A . The fit of Eq.(4) to the data points at pion mass less than 600 MeV on ensembles c005 and c02 is shown in the right graph of Fig. 1. The χ^2/dof of the fit is 0.73 and we get $m_\Delta = 1.183(64)$ GeV at the physical pion mass. This should be compared with the experiment value 1232 MeV with a width 117 MeV.

In the last column of Tab. 4 and Tab. 5, we give the mass difference between the Delta and the nucleon in lattice units. From Eq.(4), we see this mass difference is a linear function of the pion mass squared at leading order. Extrapolating the lowest three data points to the physical pion mass using a linear function of $(am_\pi)^2$, one gets $a(m_{\Delta^{++}} - m_N) = 0.155(32)$ or $m_{\Delta^{++}} - m_N = 272(56)$ MeV for ensemble c005. Similarly for ensemble c02, we get $a(m_{\Delta^{++}} - m_N) = 0.174(62)$ or

Table 5: Pion, nucleon and Delta masses from ensemble c02. $1/a = 1.75(4)$ is used to convert to physical units.

am_q	am_{ps}	m_{ps}/GeV	am_N	$am_{\Delta^{++}}$	$a(m_{\Delta^{++}} - m_N)$
0.01350	0.18440(63)	0.3227(75)	0.708(16)	0.857(97)	0.149(95)
0.02430	0.24213(53)	0.4237(97)	0.7520(91)	0.917(36)	0.165(35)
0.04890	0.33899(56)	0.593(14)	0.8272(56)	0.975(16)	0.148(20)
0.06700	0.39692(55)	0.695(16)	0.8936(41)	1.0313(94)	0.138(12)
0.15000	0.60939(47)	1.066(24)	1.1622(28)	1.2584(51)	0.0962(57)
0.33000	0.98015(30)	1.715(39)	1.6594(23)	1.7196(33)	0.0602(40)

Table 6: Pion and nucleon masses from ensemble f004. $1/a = 2.33(5)$ is used to convert to physical units.

am_q	am_{ps}	m_{ps}/GeV	am_N
0.00677	0.1134(35)	0.2642(99)	0.497(33)
0.01290	0.1550(26)	0.3612(98)	0.532(35)
0.02400	0.2106(15)	0.491(11)	0.606(44)
0.04700	0.2970(10)	0.692(15)	0.672(12)
0.18000	0.6321(10)	1.473(32)	1.0848(72)
0.28000	0.8394(11)	1.956(42)	1.3851(44)

$m_{\Delta^{++}} - m_N = 304(108)$ MeV. Both values agree with the experiment value ~ 292 MeV but have large uncertainties.

3.2 Quark correlation functions

The diquark-quark mass difference is a measure of the strength of the diquark correlation. This difference for the good diquark is estimated from hadron spectrum in Ref. [16]. We obtain the quark mass from the scalar part of the quark propagator $S(x, 0)$ in Landau gauge

$$C_q(t) = \sum_{\vec{x}} S_{ii}^{aa}(x, 0), \quad (5)$$

where the color index a and spin index i are summed over. A single exponential fit (actually a hyperbolic sine function because of the boundary condition in the time direction) taking into account data correlations to $C_q(t)$ at large t , for example $t \in [13, 28]$, gives the quark mass aM_q at each bare valence quark mass. Examples of the effective mass $\ln(C_q(t)/C_q(t+1))$ and the results from the exponential fits are shown in Fig. 2.

The quark masses aM_q are collected in Tab. 7 for all three ensembles. They are plotted against

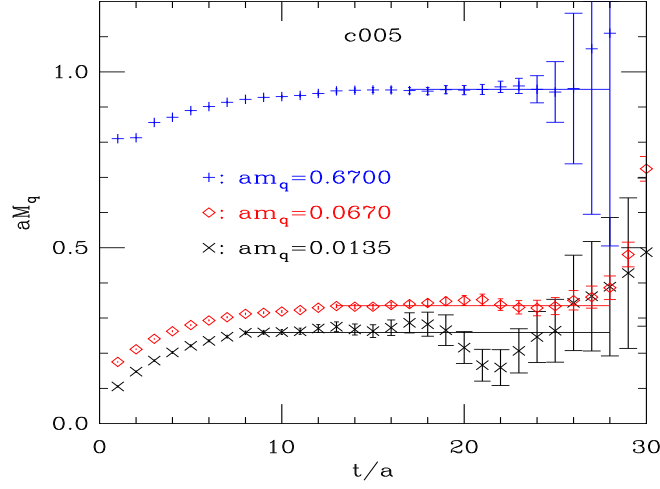


Figure 2: The effective quark mass $\ln(C_q(t)/C_q(t+1))$ at various bare valence quark masses for ensemble c005 with sea quark masses $m_l/m_s = 0.005/0.04$. The straight lines mark the results of the quark mass from single exponential fits to the correlators $C_q(t)$. The fitting ranges of t are also indicated by the lines.

Table 7: Quark masses for various valence quark masses on all three ensembles. The first line is a linear extrapolation in am_q to the chiral limit with the lowest four quark masses.

$am_q(\text{coarse})$	$aM_q(\text{c005})$	$aM_q(\text{c02})$	$am_q(\text{fine})$	$aM_q(\text{f004})$
0.0	0.2361(44)	0.2813(86)	0.0	0.1832(99)
0.01350	0.2592(47)	0.298(12)	0.00677	0.196(14)
0.02430	0.2695(35)	0.3034(73)	0.01290	0.199(10)
0.04890	0.3102(46)	0.3258(43)	0.02400	0.2144(89)
0.06700	0.3351(45)	0.3443(37)	0.04700	0.2466(81)
0.15000	0.4282(30)	0.4362(32)	0.18000	0.3893(79)
0.33000	0.6190(57)	0.6264(58)	0.28000	0.5006(92)
0.67000	0.9504(85)	0.9559(88)	0.50000	0.721(12)

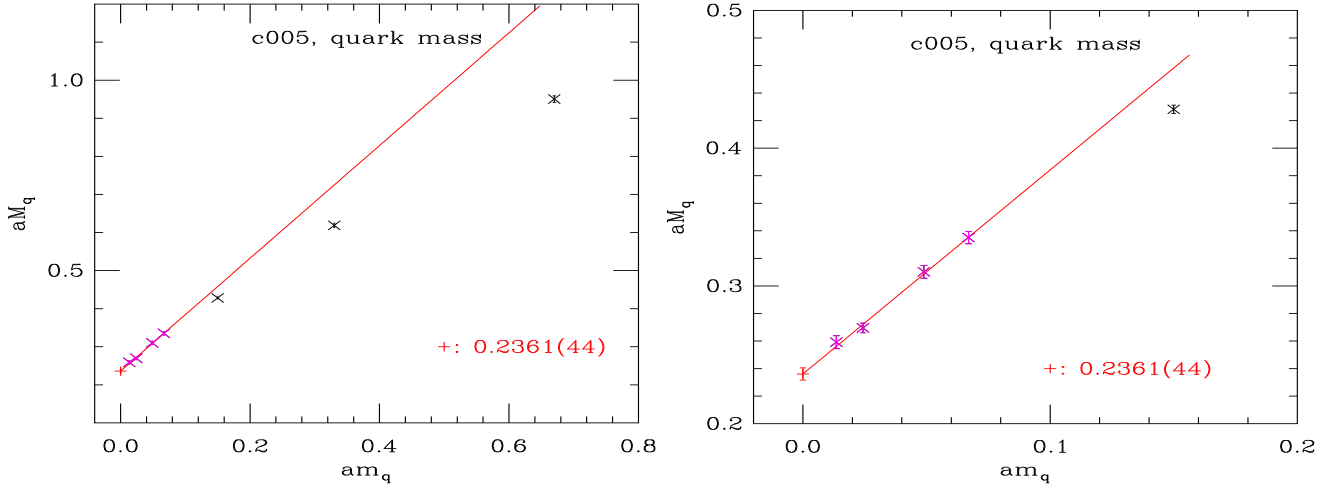


Figure 3: Quark masses aM_q against the bare valence quark mass am_q for ensemble c005 with sea quark masses $m_l/m_s = 0.005/0.04$. The straight line is a linear fit to the lightest four quark masses. The graph on the right is a zoom in of the left.

the bare quark mass for ensemble c005 in Fig. 3. With a linear fit to the lightest four quark masses (the corresponding current quark masses are not heavier than the physical strange quark mass), one finds in the chiral limit $aM_q = 0.2361(44)$ or $M_q = 413(12)$ MeV by using $1/a = 1.75(4)$ GeV. The fitting is also shown in Fig. 3 by the red line.

Similarly on the ensemble c02, we obtain $aM_q = 0.2813(86)$ or $M_q = 492(19)$ MeV in the chiral limit by a linear extrapolation with the lowest four quark masses. The two ensembles c02 and c005 have different sea quark masses. To see the sea quark mass dependence more clearly, we plot aM_q from the two ensembles together in Fig. 4. We see that there is a clear sea quark mass dependence in our results when the valence quark mass is less than the physical strange quark mass ($am_q \leq 0.067$). The quark mass aM_q from the scalar part of the quark propagator in Landau gauge decreases as the sea quark mass decreases.

The results from ensemble f004 are also given in Tab. 7. In the chiral limit of the valence quark mass, we get $aM_q = 0.1832(99)$ or $M_q = 427(25)$ MeV by using $1/a = 2.33(5)$ GeV. If using the lowest five data points to do the linear extrapolation, then we find $aM_q = 0.1889(58)$ or $M_q = 440(16)$ MeV. The light sea quark mass for ensemble f004 is lighter than but close to that for c005. Since the number $M_q = 427(25)$ MeV agrees with the result $413(12)$ MeV from c005, we do not see apparent discretization effects in M_q with our statistical uncertainty.

Although the quark mass M_q here is in principle gauge dependent, it was shown this dependence may be small in covariant gauges including the Landau gauge [25]. If we average the quark masses M_q from c005 and f004 using the inverse of their squared error as the weight, then we obtain $M_q = 416(11)$ MeV. Here the inverse of the square of the final statistical uncertainty is equal to the sum of the inverse squared error (the weight). Note our sea quark mass is still larger than the

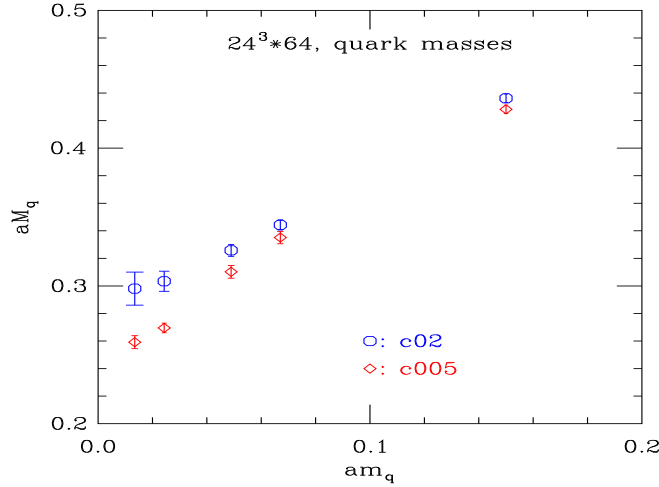


Figure 4: Quark masses from the scalar part of quark propagators against the bare valence quark masses on the two coarse lattices with different sea quark masses.

physical one. M_q is in general consistent with constituent quark masses (350-400 MeV) used in various models.

For the strange quark, its bare valence quark mass is roughly 0.067 and 0.047 on the coarse and fine lattice respectively [24]. The corresponding quark mass M_s from the scalar part of the quark propagator is 586(16), 603(15) and 575(23) MeV on the three ensembles c005, c02 and f004 respectively (see Tab. 7). Unlike the light quark mass M_q , the sea quark mass dependence in M_s is much smaller.

3.3 Diquark masses and mass differences

3.3.1 Diquarks composed of two light quarks

We start with diquarks composed of the up and down quarks, i.e. $q_1 = u$ and $q_2 = d$ in Tab. 1. The two point functions of diquarks exhibit an exponential decay behaviour similar to that of ordinary hadron correlation functions. This can be clearly seen in the effective mass plot shown by Fig. 5 from the correlation functions using J_c^5 and J_c^{05} . Both currents J_c^5 and J_c^{05} can give us the scalar diquark mass. The effective masses from the two currents go to a same plateau at large t as we see in the graph. But the plateau for J_c^{05} shows up earlier than that for J_c^5 . Therefore we fit the correlators from J_c^{05} to determine the scalar diquark mass. In Fig. 5 the lines indicate the results from single exponential fittings to the correlators at different quark masses.

The numerical results of the scalar diquark mass are given in Tab. 8 for c005. With a linear chiral extrapolation, the scalar diquark mass from J_c^{05} is 725(20) MeV by using $1/a = 1.75(4)$ GeV. Then the scalar diquark and quark mass difference from ensemble c005 is $725(20) - 413(12) =$

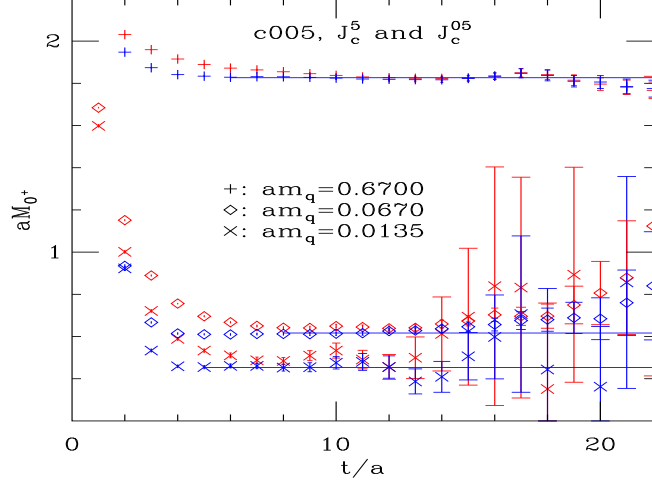


Figure 5: Effective scalar diquark masses at various valence quark masses on ensemble c005 with sea quark masses $m_l/m_s = 0.005/0.04$. The red symbols are from the correlators from J_c^5 . The blue ones from J_c^{05} . The straight lines mark the results of the scalar diquark mass from single exponential fits to the correlation functions using J_c^{05} . The fitting ranges of t are indicated by the length of the lines.

Table 8: Diquark masses and mass difference for various valence quark masses on ensemble c005. The first line is a linear extrapolation in am_q to the chiral limit with the lowest four data points.

am_q	$aM_{0+}(J_c^{05})$	$aM_{1+}(J_c^I)$	$a(M_{1+} - M_{0+})$	$aM_{0-}(J_c^I)$	aM_{1-}
0.0	0.4142(63)	0.584(21)	0.166(22)	-	-
0.01350	0.4534(70)	0.611(29)	0.158(31)	-	-
0.02430	0.4875(52)	0.635(18)	0.148(19)	0.796(52)	-
0.04890	0.5692(37)	0.694(10)	0.1248(98)	0.862(23)	0.987(53)
0.06700	0.6166(48)	0.7300(85)	0.1134(93)	0.904(18)	1.003(41)
0.15000	0.8293(70)	0.8907(68)	0.0614(89)	1.056(29)	1.140(24)
0.33000	1.1830(30)	1.2334(55)	0.0504(45)	1.378(17)	1.454(21)
0.67000	1.8265(39)	1.8604(68)	0.0339(62)	1.976(12)	2.025(16)

Table 9: Diquark masses and mass difference for various valence quark masses on ensemble c02. The first line is a linear extrapolation in am_q to the chiral limit with the lowest four data points.

am_q	$aM_{0+}(J_c^{05})$	$aM_{1+}(J_c^i)$	$a(M_{1+} - M_{0+})$	$aM_{0-}(J_c^I)$	aM_{1-}
0.0	0.4555(91)	0.644(16)	0.185(20)	-	-
0.01350	0.491(10)	0.662(21)	0.171(26)	-	-
0.02430	0.5256(80)	0.687(13)	0.161(16)	0.900(57)	-
0.04890	0.5998(75)	0.727(11)	0.127(12)	0.950(22)	0.956(79)
0.06700	0.6453(63)	0.7574(85)	0.1121(95)	0.984(16)	1.011(64)
0.15000	0.8521(76)	0.9145(58)	0.0624(87)	1.104(13)	1.165(33)
0.33000	1.2060(56)	1.2459(56)	0.0399(53)	1.400(19)	1.441(25)
0.67000	1.836(11)	1.8588(94)	0.0228(77)	1.969(14)	2.053(16)

Table 10: Diquark masses and mass difference for various valence quark masses on ensemble f004. The first line is a linear extrapolation in am_q to the chiral limit with the lowest four data points.

am_q	$aM_{0+}(J_c^{05})$	$aM_{1+}(J_c^i)$	$a(M_{1+} - M_{0+})$	$aM_{0-}(J_c^I)$	aM_{1-}
0.0	0.296(19)	0.425(24)	0.128(30)	-	-
0.00677	0.318(28)	0.438(46)	0.120(54)	-	-
0.01290	0.340(22)	0.460(24)	0.120(32)	-	-
0.02400	0.379(15)	0.487(18)	0.108(23)	-	-
0.04700	0.457(10)	0.547(15)	0.090(18)	0.61(10)	0.565(17)
0.18000	0.7879(84)	0.835(11)	0.047(13)	0.957(20)	0.829(19)
0.28000	0.9977(84)	1.031(10)	0.033(11)	1.136(26)	1.017(13)
0.50000	1.4299(90)	1.447(14)	0.017(15)	1.592(16)	1.454(15)

312(23) MeV. Here the final uncertainty is from a simple error propagation. This number is in good agreement with the estimation ~ 310 MeV in Ref. [16].

The results from the ensemble c02 are obtained similarly and are collected in Tab. 9. In the chiral limit, the scalar diquark mass from ensemble c02 is 797(24) MeV by using $1/a = 1.75(4)$ GeV. This value is a little different from 725(20) MeV for ensemble c005. Thus there seems to be some sea quark mass dependence in the absolute value of the scalar diquark mass. The scalar diquark and quark mass difference is $797(24) - 492(19) = 305(31)$ MeV, which agrees with the result 312(23) MeV from ensemble c005. Therefore we do not see sea quark mass dependence in the diquark quark mass difference.

The results on the fine lattice f004 are given in Tab. 10. The scalar diquark mass in the chiral limit is $aM_{0+} = 0.296(19)$ or $M_{0+} = 690(47)$ MeV. With a relatively large error, it is in agreement with the number 725(20) MeV from ensemble c005, which indicates the finite lattice spacing effect is smaller than our statistical uncertainty. The mass difference between the scalar diquark and

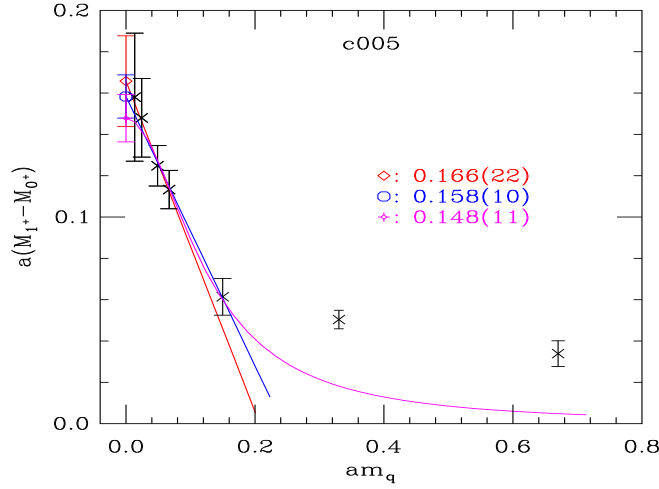


Figure 6: The mass difference between bad and good diquarks as a function of the valence quark mass on ensemble c005. The diquarks are composed of two degenerate light quarks. The red straight line is a linear extrapolation in am_q to the chiral limit using the lowest four data points. The blue straight line uses the lowest five data points. The magenta curve is the fit using Eq.(6) to the lowest five points.

the light quark is $690(47) - 427(25) = 263(53)$ MeV. It just agrees with the results $312(23)$ MeV and $305(31)$ MeV from c005 and c02 respectively.

We average the scalar diquark and quark mass difference from the three ensembles using the inverse of their squared error as the weight. The sum of the inverse squared error gives the inverse of the final uncertainty squared. In this way, we obtain $M_{0+} - M_q = 304(17)$ MeV, where the error is statistical (including the uncertainties of lattice spacing).

The mass of the bad diquark can be extracted from the correlators using J_c^i or J_c . The fitting results by using a single exponential from the two currents are in agreement. However the signal of the correlator from J_c^i is better. Therefore we collect the bad diquark masses from this current in Tab. 8 for c005. A linear extrapolation to the chiral limit with the lowest four data points gives $aM_{1+} = 0.584(21)$ or $M_{1+} = 1022(44)$ MeV.

The bad and good diquark mass difference (in lattice units) is plotted against the valence quark mass in Fig. 6. Here the uncertainties are from bootstrap analysis. As we can see, the difference decreases as the quark mass increases. The simplest chiral extrapolation of the mass difference is a straight line fit. Using the lowest four data points, one gets $a(M_{1+} - M_{0+}) = 0.166(22)$ or $M_{1+} - M_{0+} = 291(39)$ MeV. If we use the lowest five data points for the linear chiral extrapolation, then we get $a(M_{1+} - M_{0+}) = 0.158(10)$, which agrees with the result by using the lowest four data points.

We also tried a ansatz similar to the one used in Ref. [7]. Diquark correlations come from spin dependent forces. The bad and good diquark mass difference Δm is expected to scale like

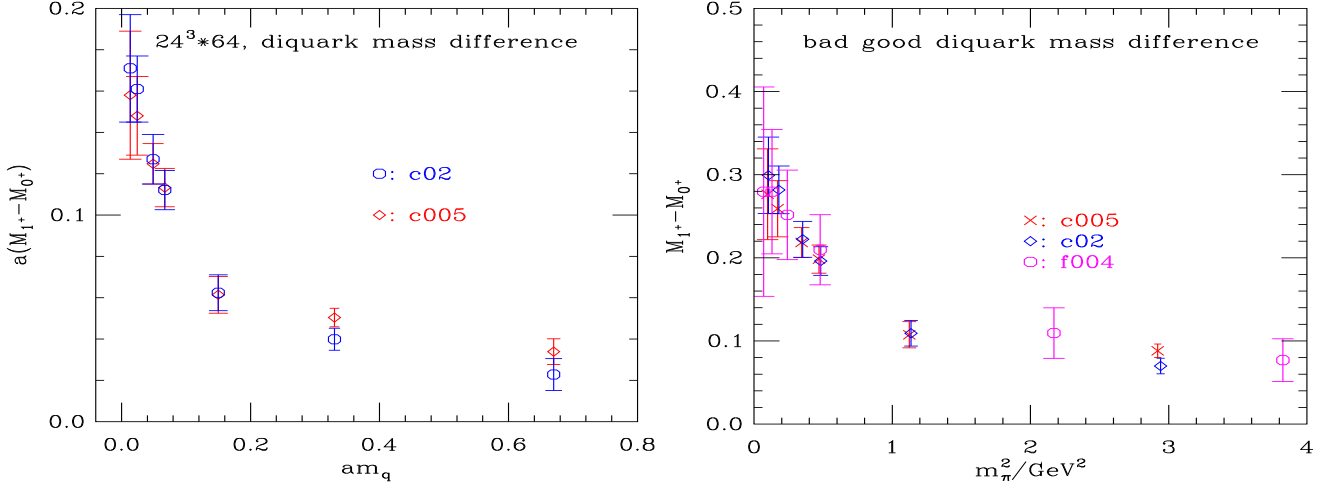


Figure 7: Left: The mass difference between bad and good diquarks as a function of the valence quark mass on ensemble c005 and c02. The results from the two ensembles are in agreement, which means the mass difference has small sea quark mass dependence. Right: The mass difference (GeV) against the pion mass squared on all three ensembles.

$1/(m_{q_1}m_{q_2})$ at large quark mass [16]. In the chiral limit, the mass difference goes to a constant. Thus one can try the following ansatz for diquarks composed of two degenerate quarks

$$\Delta m = \frac{a_1}{1 + a_2 m_q^2}, \quad (6)$$

where a_1 and a_2 are two fitting parameters. We find that this ansatz can not fit all seven data points with an acceptable χ^2/dof . If we limit to the lowest five data points, we can get a $\chi^2/\text{dof} < 1$ and find $a(M_{1+} - M_{0+}) = 0.148(11)$. The fit is shown by the curve in Fig. 6.

For the bad diquark from ensemble c02, a linear extrapolation to the chiral limit with the lowest four data points gives $aM_{1+} = 0.644(16)$, which is higher than $0.584(21)$ from ensemble c005. The bad and scalar diquark mass difference is given in the fourth column of Tab. 9. Unlike the absolute value of diquark masses, the diquark mass differences on the two ensembles are in agreement within statistical uncertainties. In the left graph of Fig. 7, the mass differences are plotted against the valence quark mass from ensembles c005 and c02. It does not show apparent sea quark mass dependence.

On ensemble f004, the bad diquark mass in the chiral limit is $aM_{1+} = 0.425(24)$ or $M_{1+} = 990(60)$ MeV, which is in agreement with the result $1022(44)$ MeV from c005. Therefore we do not see discretization effects with our current statistical uncertainties. The fourth column in Tab. 10 is the bad and good diquark mass difference on the fine lattice.

We plot the bad and good diquark mass difference in physical units against the pion mass squared on all three ensembles in the right graph of Fig. 7. All lattice results seem to lie on

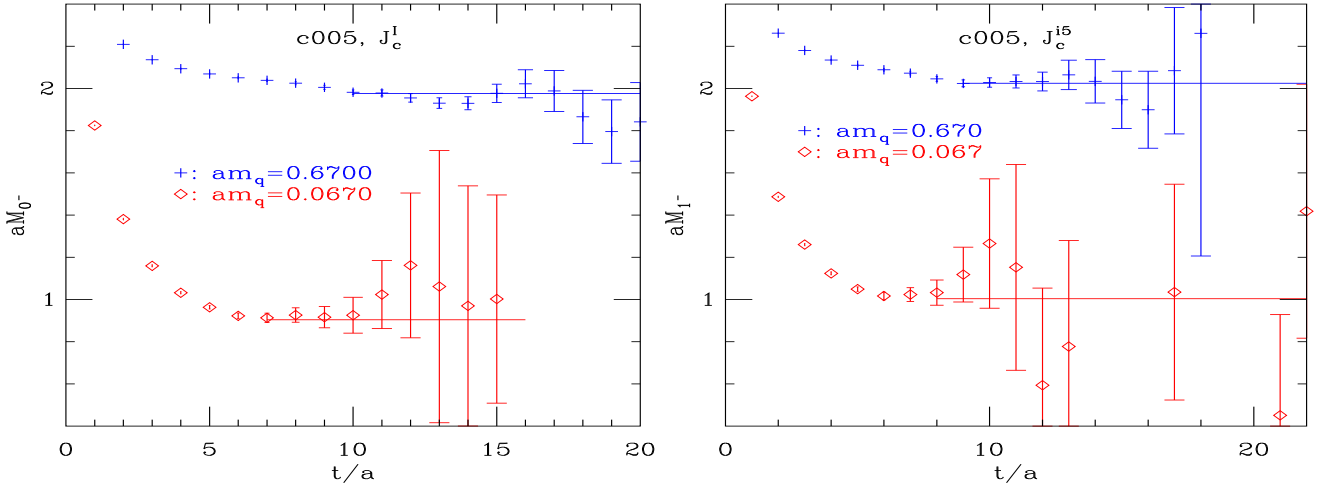


Figure 8: Effective masses of the 0^- (left) and 1^- (right) diquarks at two valence quark masses on ensemble c005. The 0^- diquark uses correlators from J_c^I . The straight lines mark the diquark mass from single exponential fits to the correlation functions. The fitting ranges of t are indicated by the length of the lines.

a universal curve, which means sea quark mass dependence and discretization effects are small compared with the statistical errors. By using the lowest order relation $m_\pi^2 \propto m_q$, Eq.(6) can be written as

$$\Delta m = \frac{b_1}{1 + b_2 m_\pi^4}. \quad (7)$$

This ansatz can fit the data points at $m_\pi^2 < 1.2(0.6) \text{ GeV}^2$ and gives $M_{1+} - M_{0+} = 264(14)(285(20)) \text{ MeV}$ ($\chi^2/\text{dof}=3.2/12(1.2/10)$). Alternatively, a linear extrapolation in m_π^2 with the data points at $m_\pi^2 < 1.2(0.6) \text{ GeV}^2$ gives $M_{1+} - M_{0+} = 280(12)(309(25)) \text{ MeV}$. The average of the four center values is 285 MeV. Taking the largest statistical error and the largest change in the center value as the systematic error, we get $M_{1+} - M_{0+} = 285(25)(45) \text{ MeV}$. This number is a little bigger than the estimation $\sim 210 \text{ MeV}$ in Ref. [16], which used masses of baryons with a strange or charm quark.

For the diquark with quantum number $J^P = 0^-$, we tried two interpolating operators J_c^I and J_c^0 . The correlators from both operators are noisy with the one from J_c^I having a better signal. The masses from single exponential fits to the correlators of J_c^I on ensemble c005, c02 and f004 are given in Tabs. 8, 9, 10 respectively. Examples of the effective mass plateau are shown in the left graph of Fig. 8. At some of the small valence quark masses, no result is obtained due to the bad signal to noise ratio. These results confirm that the 0^- diquark is heavier than both the 0^+ and 1^+ diquarks.

The correlator for the vector diquark is even noisier. The extracted diquark masses are listed in Tab. 8 and Tab. 9 for the two ensembles on the coarse lattice respectively. The right graph in

Table 11: Diquark masses and mass difference for various light quark masses on ensemble c005. The diquarks are composed of a strange and a light quark. The first line is a linear extrapolation in am_{q_1} to the chiral limit.

am_{q_1}	$aM_{0+}(J_c^{05})$	$aM_{1+}(J_c^i)$	$a(M_{1+} - M_{0+})$
0.0	0.5177(43)	0.609(16)	-
0.01350	0.5376(42)	0.633(18)	0.095(18)
0.02430	0.5534(38)	0.656(14)	0.103(15)
0.04890	0.5884(32)	0.691(10)	0.103(11)
0.06700	0.6166(48)	0.7300(85)	0.1134(93)

Table 12: Diquark masses and mass difference for various light quark masses on ensemble c02. The diquarks are composed of a strange and a light quark. The first line is a linear extrapolation in am_{q_1} to the chiral limit.

am_{q_1}	$aM_{0+}(J_c^{05})$	$aM_{1+}(J_c^i)$	$a(M_{1+} - M_{0+})$
0.0	0.579(13)	0.6883(86)	-
0.01350	0.593(14)	0.7037(89)	0.111(17)
0.02430	0.602(12)	0.7119(72)	0.110(14)
0.04890	0.629(10)	0.7337(60)	0.105(12)
0.06700	0.6453(63)	0.7574(85)	0.1121(95)

Fig. 8 shows two examples of the effective mass for this channel. The vector diquark seems to be heavier than the 0^- diquark. But with our statistical uncertainty, it is hard to determine. On ensemble f004 (see Tab. 10), the 0^- diquark seems to be heavier than the vector diquark. More statistics are needed to improve the mass plateaus for the 0^- and 1^- diquarks.

3.3.2 Diquarks with a strange and a light quark

Now we turn to diquarks composed of a strange and a light quark. We set $q_1 = u$ and $q_2 = s$ in the currents given in Tab. 1. Therefore we use $am_{q_2} = 0.0670$ and $am_{q_1} = 0.0135, 0.0243, 0.0489, 0.0670$ on the coarse lattice. The scalar and bad diquark masses together with their difference are given in Tabs. 11, 12 as we vary the mass of q_1 on the two ensembles c005 and c02.

In the chiral limit of the up quark, we obtain $aM_{0+} = 0.5177(43)$ by doing a linear extrapolation in am_{q_1} on ensemble c005. In physical units, it is 906(22) MeV. For the scalar diquark and strange quark mass difference, one gets $0.5177(43) - 0.3351(45) = 0.1826(62)$ in lattice units or 320(13) MeV, which is of the same size as 312(23) MeV for the scalar diquark composed of two light quarks. On ensemble c02, this difference is $0.579(13) - 0.3443(37) = 0.235(14)$ or 411(26) MeV. It is heavier than the result on c005, showing some sea quark mass dependence. This dependence

Table 13: Diquark masses and mass difference for various light quark masses on ensemble f004. The diquarks are composed of a strange and a light quark. The first line is a linear extrapolation in am_{q_1} to the chiral limit.

am_{q_1}	$aM_{0+}(J_c^{05})$	$aM_{1+}(J_c^i)$	$a(M_{1+} - M_{0+})$
0.0	0.3862(93)	0.453(14)	-
0.00677	0.395(11)	0.466(16)	0.071(19)
0.01290	0.406(10)	0.481(15)	0.075(17)
0.02400	0.4246(97)	0.497(13)	0.072(17)
0.04700	0.457(10)	0.547(15)	0.090(18)

mainly comes from the scalar diquark mass since the strange quark mass M_s is not so sensitive to the light sea quark mass (see the end of Sec. 3.2).

On the fine lattice, we set $am_{q_2} = 0.04700$ and $am_{q_1} = 0.00677, 0.01290, 0.02400, 0.04700$. In Tab. 13, we give the diquark masses from ensemble f004. The scalar diquark and strange quark mass difference is $0.3862(93) - 0.2466(81) = 0.140(12)$ or $326(29)$ MeV. It agrees with the result $320(13)$ MeV from c005, indicating small discretization effect in this difference. Averaging the results from c005 and f004 weighted by their inverse squared error, one gets $M_{0+} - M_s = 321(12)$ MeV for diquarks composed of a light and a strange quark.

Using the results 320 MeV and 411 MeV from c005 and c02 respectively, we can do a linear extrapolation to the light sea quark massless limit: $a(m_{sea} + m_{res}) = 0$. What we get is $271(49)$ MeV. Taking the difference between 320 MeV and 271 MeV as a systematic error, we find $M_{0+} - M_s = 321(12)(49)$ MeV. This number is smaller than the estimation ~ 500 MeV in Ref. [16].

For the bad diquark mass, we also do a linear extrapolation in the light valence quark mass and find $0.609(16)$ in the chiral limit ($1.066(37)$ MeV in physical units) on c005. Using this number and the chiral limit value of the good diquark mass $0.5177(43)$, one gets the mass difference in the chiral limit as $0.091(17)$ or $159(30)$ MeV. It agrees with the estimation of this diquark mass difference (152 MeV) in Ref. [16] obtained from baryon masses in the charm sector. Compared with the case for diquarks composed of two light quarks, this difference decreases as one of the light quark is changed to a strange quark.

On ensemble c02, the absolute values of the bad and good diquark masses seem heavier than their counterparts on ensemble c005, indicating some sea quark mass dependence. This is similar to the case for diquarks composed of two light quarks. The mass difference between the bad and good diquarks on ensemble c02 agrees with that on ensemble c005, showing that the sea quark mass dependence is smaller in the difference than in the absolute diquark masses.

On ensemble f004, the scalar and bad diquark masses in the chiral limit of m_{q_1} are $900(29)$ MeV and $1.055(40)$ MeV respectively. Both are in agreement with their counterparts from c005. Thus discretization effects are again shown to be small.

The scalar and bad diquark mass difference from all three ensembles are plotted in Fig. 9 as

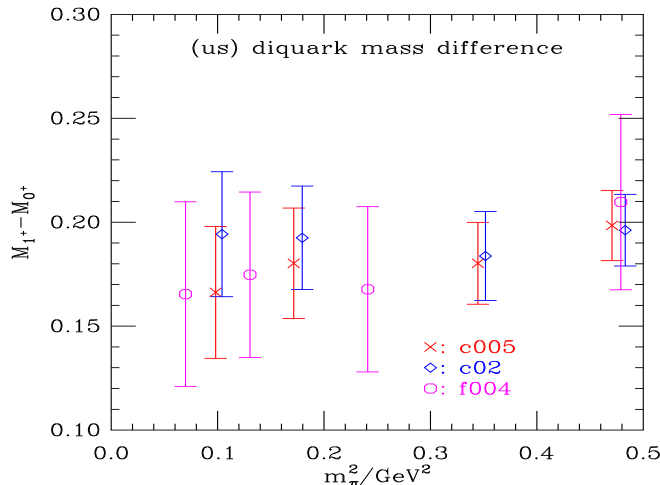


Figure 9: The scalar and bad diquark mass difference (GeV) against the pion mass squared on all three ensembles. The diquarks are composed of a strange and a light quark.

a function of the squared mass of the pion composed of light quark q_1 (see Tabs. 4, 5, 6). As we can see from the graph, the dependence of the mass difference on the light quark mass (or pion mass) seems small with our current statistical uncertainty. One can fit the difference either with a straight line in m_{π}^2 or with a constant. The straight line fit gives $M_{1+} - M_{0+} = 171(18)$ MeV. The constant fit gives $M_{1+} - M_{0+} = 188(7)$ MeV. Averaging the two, we get $180(18)(17)$ MeV. Here the first uncertainty is the bigger one of the two statistical errors and the second one is the systematic error from the change in the center values.

4 Summary

Using overlap valence quark on configurations with 2+1 flavors of domain wall sea quarks, we calculated the mass and mass difference of various diquarks in the Landau gauge. Also the diquark quark mass difference is computed. We extrapolate the results to the valence quark chiral limit and check the sea quark mass dependence using two ensembles with a same coarse lattice spacing. Discretization effects are examined by working on a fine lattice.

The scalar diquark has the lowest mass which means it is the channel with the strongest correlation. The mass difference between the axial vector and scalar diquark (composed of two light quarks) decreases as the valence quark mass increases. This was also observed in previous lattice calculations [6, 7, 8].

We see sea quark mass dependence in the absolute values of diquark and quark masses. Their masses decrease as the sea quark mass decreases. This dependence is small (smaller than our statistical error) in diquark mass difference and diquark quark mass difference. For the diquark

Table 14: Diquark mass difference and diquark quark mass difference (MeV). The diquarks are either composed of two light quarks or composed of a strange and a light quark. The first (or the only) error is statistical and the second (when there is one) is a systematic error.

(ud)	$M_{0+} - M_q$	$M_{1+} - M_{0+}$
this work	304(17)	285(25)(45)
[16]	~ 310	~ 210
(us)	$M_{0+} - M_s$	$M_{1+} - M_{0+}$
this work	321(12)(49)	180(18)(17)
[16]	~ 500	~ 150

composed of a strange and a light quark, the mass difference between the scalar diquark and the strange quark shows some sea quark mass dependence. From our data we do not expect this difference to increase as the light sea quark mass lowers to the physical value. Within our limited statistics on the fine lattice, we do not see apparent discretization effects in all our results.

Our final results of the mass differences are given in Tab. 14. In the chiral limit of the valence quark mass, We find the diquark mass difference $M_{1+} - M_{0+} = 285(25)(45)$ MeV and diquark quark mass difference $M_{0+} - M_q = 304(17)$ MeV for diquarks composed of two light quarks. For diquarks composed of a strange and a light quark, we obtain $M_{1+} - M_{0+} = 180(18)(17)$ MeV and $M_{0+} - M_s = 321(12)(49)$ MeV. Here when there are two uncertainties, the first one is statistical and the second one is a systematic error estimated from different extrapolations to the chiral limit or from light sea quark mass dependence. In general, the results of these mass differences agree with the estimations from hadron spectroscopy in Ref. [16]. The exception is $M_{0+} - M_s = 321(12)(49)$ MeV for the scalar diquark composed of a strange and an up quark, which is smaller than the estimation ~ 500 MeV in Ref. [16].

To better control the light sea quark mass dependence and finite lattice spacing effects in our work, calculations at another sea quark mass are needed, and more statistics on the fine lattice should be added. It might be interesting to calculate these differences in other gauges to check the gauge dependence.

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